No: ________________________

IMPERIAL COLLEGE LONDON

Design Engineering MEng EXAMINATIONS 2024

For Internal Students of the Imperial College of Science, Technology and Medicine
This paper is also taken for the relevant examination for the Associateship or Diploma

DESE50002 – Electronics 2

Date: 1 May 2024 (one hour thirty minutes)

This paper contains SIX questions.
Attempt ALL questions.

The numbers of marks shown by each question are for your guidance only; they indicate how the examiners intend to distribute the marks for this paper.

This is a CLOSED BOOK Examination.

1. a) (i) Sketch in the answer book the signal $x_0(t) = u(t) - u(t-2)$.

[2]

(ii) Sketch in the answer book the signal $x_1(t) = t \times [u(t) - u(t-2)]$.

[4]

(iii) State the equation that describes the signal $y(t)$ shown in ***Figure Q1***.

[6]

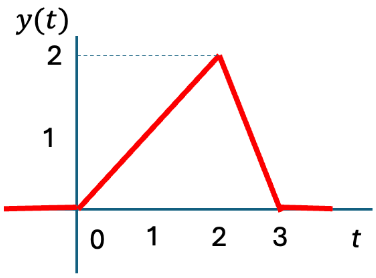


Figure Q1

b) A signal $x(t)$ is mathematically modelled by the following equation where $\delta(t)$ is the unit impulse function. Sketch in your answer book the signal $x(t)$.

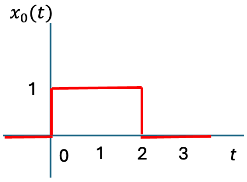
$$x(t) = 5\,\delta(t+2) + 2\delta(t-2) - 5\delta(t-1) + \delta(t)$$

[3]

Solution to Q1

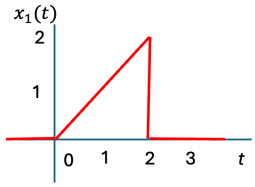
This question tests student's ability to model signal mathematically in terms of $u(t)$ and $\delta(t)$ and the application of the shift property of signals.

a) (i) $x_0(t) = u(t) - u(t-2).$



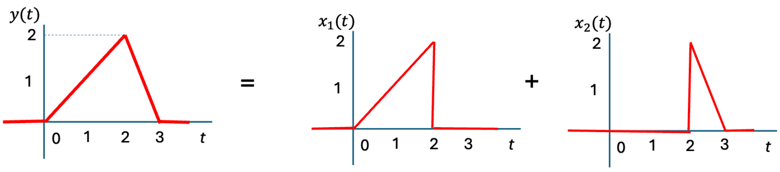
[2]

(ii) $x_1(t) = t \times [u(t) - u(t-2)]$

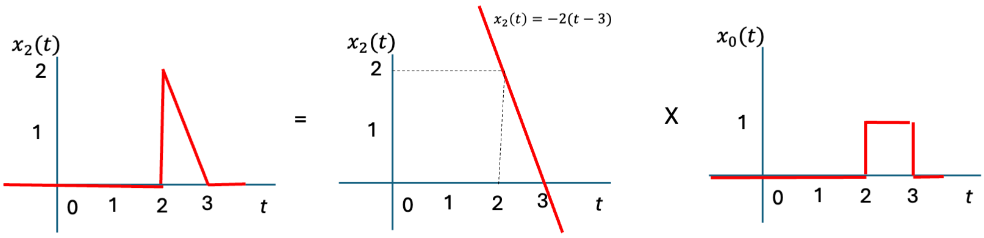


[4]

(iii) $y(t)$ can be model by $y(t) = x_1(t) + x_2(t)$, as shown below.



The signal $x_2(t)$ can be obtained by multiplying another ramp by the gate pulse as shown below. This ramp has a slope −2; hence it can be described by $x_2(t) = -2t + c$. Now, because the ramp has a zero value at t = 3, the constant c = 6. Therefore, the ramp $x_2(t)$ can be described by:

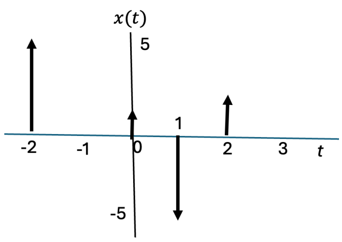


Hence, $y(t) = x_1(t) + x_2(t) = t[u(t) - u(t-2)] - 2(t-3)[u(t-2) - u(t-3)]$

$$= tu(t) - 3(t-2)u(t-2) + 2(t-3)u(t-3)$$

[6]

b)



[3]

2. A signal $y(t)$ is mathematically modelled by the following equation:

$$y(t) = 2.0 \times \cos\left(628.32t + \frac{\pi}{2}\right) + 1.0$$

a) Rewrite $y(t)$ in exponential form.

[3]

b) The signal is sampled at a sampling frequency of 400 Hz. What are the amplitude values of $y[n]$ for $n = 0$ to 5?

[4]

c) Write in the answer book the equation that models the discrete signal $y[n]$ at the sampling frequency of 400 Hz.

[4]

d) A rectangular window is applied to the signal $y[n]$ to form a new signal $w[n]$ such that:

$$w[n] = \begin{cases} y[n] & \text{for } 0 \le n \le 5 \\ 0 & \text{otherwise} \end{cases}.$$

Write in the answer book the equation that describes the windowed signal $w[n]$.

[4]

Solution to Q2

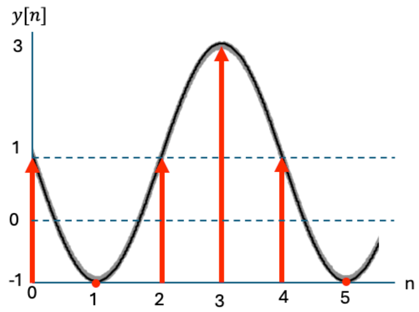
This question tests student's understanding of the relationship between discrete and continuous time signals, and the exponential form of a sinusoidal signal.

(a)

$$y(t) = \left(e^{j(628.32t+\frac{\pi}{2})} + e^{-j(628.32t-\frac{\pi}{2})}\right) + 1.0$$

[3]

(b) A phase shift of $\frac{\pi}{2}$ of a cosine function is shifting the cosine wave to the left by $90°$. This becomes a negative sine function with zero phase shift at the same frequency of 628.32 rad/sec = 100 Hz. Therefore, if sampled at 400Hz, there are 4 samples per cycle as shown below.



Hence the values are: [1, -1, 1, 3, 1, -1].

[4]

(c) With signal frequency at 100Hz and sampling frequency of 400Hz, each sampling interval will increase the angle by $\frac{\pi}{2}$. Hence the equation is:

$$y[n] = 2.0 \times \cos\left(\frac{\pi}{2}n + \frac{\pi}{2}\right) + 1.0 = 2.0 \times \cos\left(\frac{\pi}{2}(n+1)\right) + 1.0 \text{ CC}$$

[4]

(d) Based on the answer in (b), the equation is:

$$w[n] = \delta(t) - \delta(t-2) + \delta(t-3) + 3\delta(t-4) - \delta(t-5)$$

[4]

3. ***Figure Q3a*** shows an electronic circuit that produces an output $y(t)$ from two input signals $x_1(t)$ and $x_2(t)$, where $x_1(t) = 1 + \sin(2\pi f_1 t)$ and $x_2(t) = \sin(2\pi f_2 t)$. Furthermore, it is known that $f_1 \ll f_2$.

(a) Assume that the electronic circuit is a summing amplifier that adds the two input signals to produce the output. Sketch in the answer book the two-sided exponential Fourier spectrum of the output signal $y_1(t)$. That is:

$$y_1(t) = x_1(t) + x_2(t).$$

[4]

(b) Assume that the electronic circuit is a multiplier that multiplies the two input signals to produce the output. Sketch in the answer book the two-sided exponential Fourier spectrum of the output signal $y_2(t)$. That is:

$$y_2(t) = x_1(t) \times x_2(t).$$

(Hint: Multiplication in the time domain is convolution in the frequency domain.)

[6]

(c) The input signal $x_1(t)$ to the multiplier is replaced with a signal source whose spectral profile $Y_1(f)$ is as shown in ***Figure Q3b***, where the maximum frequency component of the signal is f_m. The other input $x_2(t)$ remains unchanged.

Sketch in the answer book the two-sided exponential Fourier spectrum of the output signal $y_2(t)$.

[6]

(d) The output signal $y_2(t)$ from part (c) is to be sampled into discrete signals at a sample frequency f_{samp}.

Assume that f_m= 4kHz and f_2 = 20kHz, what is the minimum value of f_{samp} to ensure that no aliasing occurs? Briefly justify your answer.

[4]

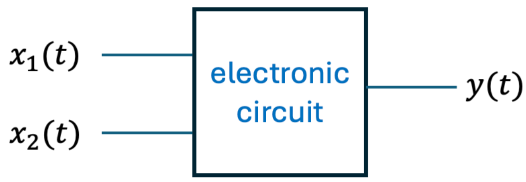


Figure Q3a

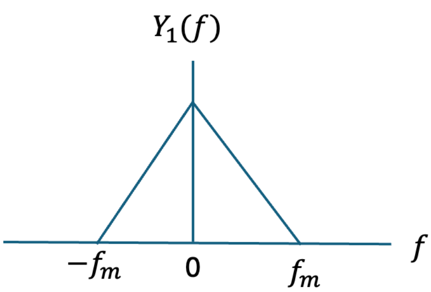
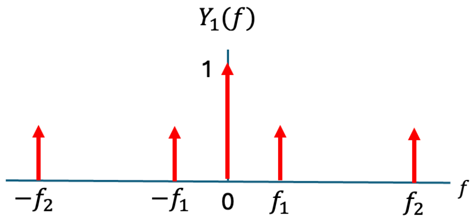


Figure Q3b

Solution to Q3

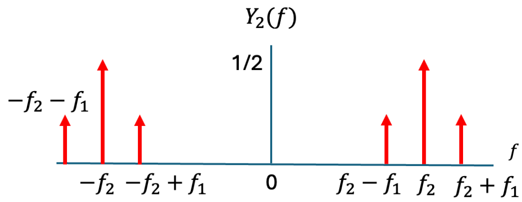
This question tests student's understanding frequency spectra, convolution and sampling theorem.

(a)



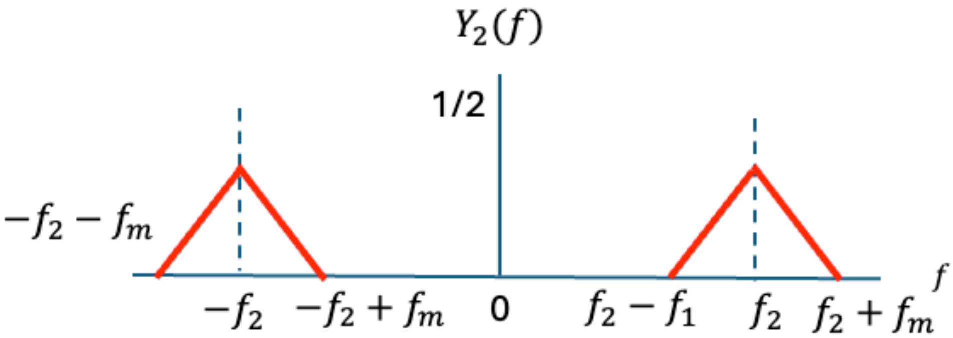
[4]

(b) Spectrum of $y_2(t)$ is the convolution of the spectrum of the two input signals. Hence,



[6]

(c) Spectrum of $y_2(t)$ is similar to (b), but the spectrum now has a triangular shape on both sides.



[6]

(d) The maximum signal frequency is $f_2 + f_m$. Therefore, according to Nyquist sampling theorem, our minimum sampling frequency f_{samp} is 2 x 24kHz = 48kHz.

[4]

4) The denominator of the transfer function of a simple second-order system is given by:

$$D(s) = s^2 + 2\zeta\omega_0 s + {\omega_0}^2 \ .$$

A simplified one-dimensional model of an automobile suspension system is shown in ***Figure Q4***. The input is the displacement $x(t)$ of the road surface from a reference ground elevation. The output is the distance $y(t)$ of the car body from the same ground reference. M is the mass of the vehicle, K is the spring constant and B is the damper constant.

If the differential equation relating the output $y(t)$ to the input $x(t)$ is given by:

$$Ky(t) + B\frac{dy(t)}{dt} + M\frac{d^2y(t)}{d^2t} = Kx(t) + B\frac{dx(t)}{dt}$$

a) Derive the transfer function $H(s) = Y(s)/X(s)$.

[10]

b) What is the system's DC gain?

[2]

c) What is the natural frequency of the system?

[4]

d) What is the damping ratio of the system?

[4]

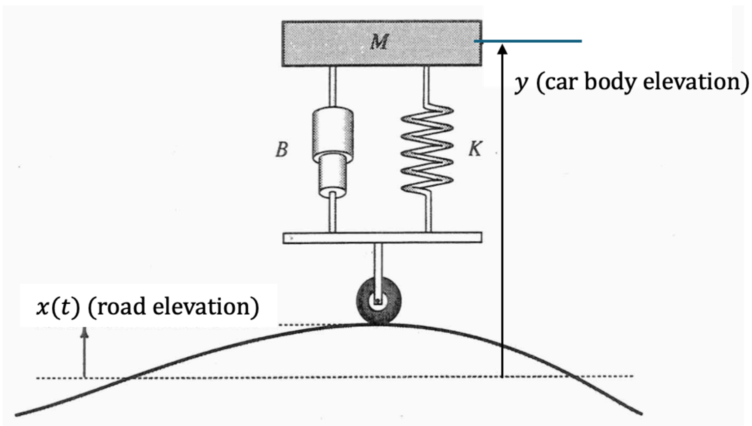


Figure Q4

Solution to Q4

This question tests student's understanding Laplace transform and its relationship to differential equation for describing behaviour of a system, transfer function, and 2nd order system characteristics, natural frequency, and damping ratio.

(a)

$$KY(s) + BsY(s) + Ms^2Y(s) = KX(s) + BsX(s)$$

$$(K + Bs + Ms^2)Y(s) = (K + Bs)X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{K + Bs}{K + Bs + Ms^2} = \frac{1 + s\frac{B}{K}}{1 + s\frac{B}{K} + s^2\frac{M}{K}}$$

[10]

(b) The DC gain = 1.

[2]

(c) Equate the denominator of this transfer function and that of a generic 2nd order system:

$$s^2 + 2\zeta\omega_0 s + {\omega_0}^2 = 1 + s\frac{B}{K} + s^2\frac{M}{K} = \frac{M}{K}(s^2 + \frac{B}{M}s + \frac{K}{M})$$

Therefore, $\quad {\omega_0}^2 = \frac{K}{M}, \omega_0 = \sqrt{\frac{K}{M}}$

[4]

(d)

$$4\zeta^2\omega_0^2 = 4\zeta^2\frac{K}{M} = \frac{B^2}{M^2},$$

Hence,

$$\zeta^2 = \frac{B^2}{4KM} \qquad \zeta = \frac{1}{2}\frac{B}{\sqrt{KM}}.$$

[4]

5. A digital filter has an impulse response $h[m]$ as shown in ***Figure Q5a***.

a) What is the transfer function $H[z]$ of this filter?

[3]

b) A signal $x[n]$ shown in ***Figure Q5b*** is applied to the input of the filter. Write down the difference equation which relates the output signal $y[n]$ of the filter to its input $x[n]$.

[4]

c) Derive the output $y[n]$ for $0 \le n \le 7$.

[8]

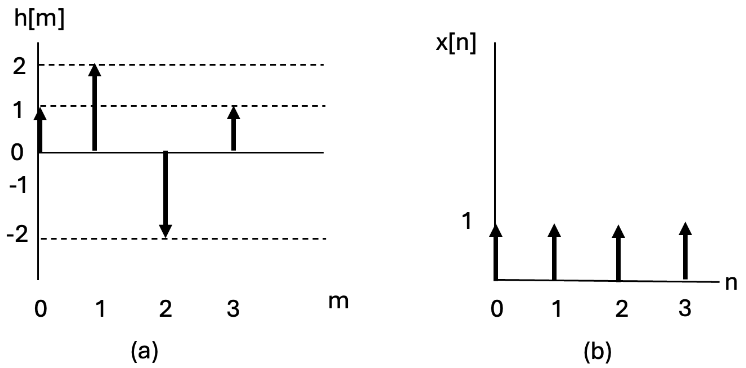


Figure Q5

Solution to Q5

This question tests student's understanding of discrete signals, impulse response of discrete system and discrete convolution.

(a)

$$H(z) = 1 + 2z^{-1} - 2z^{-2} + z^{-3}$$

[3]

(b)

$$y[n] = x[n] + 2x[n-1] - 2x[n-2] + x[n-3]$$

[4]

(c) Assume that y[n] and x[n] = 0, for n<0.

n	0	1	2	3	4	5	6	7
y[n]	1	3	1	2	1	-1	1	0

[8]

6. ***Figure Q6*** shows an open-loop process with transfer function:

$$H(s) = \frac{1}{s^2 + 5s + 6}$$

(a) Sketch in the answer book a diagram showing the system in a feedback control loop by adding a proportional-only controller to the system with a proportional gain of 5.

[5]

(b) Derive the closed-loop transfer function with input $R(s)$ and output $Y(s)$.

[5]

(c) Explain how the controller design can be improved. State the benefit and drawbacks that such modifications may bring to the system's dynamic and static behaviour.

[5]

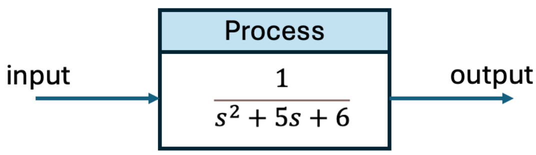
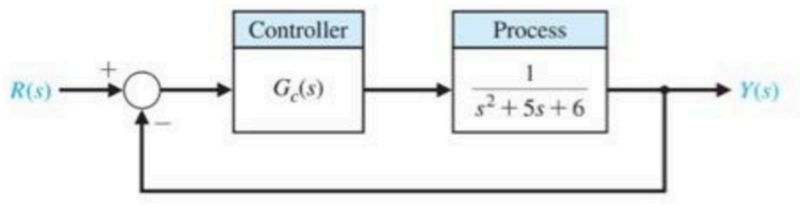


Figure Q6

Solution to Q6

This question tests student's understanding of open-loop and closed-loop system, feedback control, and PID controller.

(a)



$G_c(s) = K_p$ where K_p = 5.

[5]

(b)

$$H_{cl}(s) = \frac{Y(s)}{R(s)} = \frac{5}{s^2 + 5s + 11}$$

[5]

(c) Book work. Keywords and concept shown in bold.

The controller can be improved by adding an **integral term** which acts like a memory for the controller. It **remembers past errors** and continuously adjusts the controller's output to **eliminate the steady-state error**, ensuring the system eventually **reaches the desired setpoint** without an offset. However, the integral term may cause the system to **overshoot**.

The controller can also be improved by adding a **differential term**. This can å**improve the response speed** and **stability** of the control system by reacting to the **rate of change of the error** between the setpoint and the output. It can also reduce the overshoot. However, the differential term increases the system's **sensitivity to noise** introduced to the system (e.g. on the sensors).

[5]

[END OF PAPER]